An Application of Gamma-Poisson Model to Estimate Turbojet Engine In-Flight Shutdown Rate

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Abstract

It is of interest to determine whether an air operator is experiencing an engine in-flight shutdown event at an extraordinary high rate due to causes such as severe operating conditions, poor maintenance programs, etc. This requires an estimate of the engine in-flight shutdown rate of the air operator being considered and other air operators that use similar engines. We use the maximum likelihood estimate (MLE), hierarchical Bayes (HB), and a parametric empirical Bayes (PEB) approximation to estimate the engine in-flight shutdown rate for each combination of air operator and engine group and compare the results. We conclude that the HB and PEB approximation are preferred over the MLE for estimating engine in-flight shutdown rate over time in that they provide nonzero estimates with less variability compared with the MLE estimate, and the PEB approximation is preferred over the HB since it can be more easily automated.

Keywords: Hierarchical Bayes, Parametric empirical Bayes, Engine in-flight shutdown rate, Maximum likelihood estimate, Conjugate gamma distribution

1. Introduction

Although engines are built very reliable, there is a chance they still might fail during the flight. This engine failure event is referred to as an engine in-flight shutdown in the aviation community. The engine in-flight shutdown event data can be used to determine the engine's in-flight shutdown rate from two different perspectives: from the overall engine's in-flight shutdown rate regardless of the air operator and in concert with the air operator. The objective of the latter, which is the focus of this paper, is to determine whether or not an air operator is experiencing engine in-flight shutdown events at an extraordinary high rate due to causes such as severe operating conditions, poor maintenance programs, etc. The true engine in-flight shutdown rate is the rate at which the engine fails during the flight and is measured in terms of the number of engine shutdowns per 1000 engine hours.

The engine in-flight shutdown event is assumed to be completely random since the probability of having an engine in-flight shutdown in a small interval is considered independent of the past and depends only on the length of the interval and the true engine in-flight shutdown rate. Furthermore, the probabilities of engine in-flight shutdowns in different time intervals are considered independent. The aforementioned assumptions are developed based on the fact that an "engine" is indeed just the serial number. The engine's parts are continually replaced over time; some parts are removed at a certain number of engine hours and some are replaced during overhaul or inspections if necessary. We, therefore, assume that engines do not age; however, more efficient engines built by newer technology can replace them over time.

Thus, we can assume that the number of engine in-flight shutdowns, X_{ijk}, during the total engine hours, t_{ijk}, follow a Poisson distribution with parameter $\lambda_{ijk} t_{ijk}$, where *i*, *j*, and *k* represent the coded air operator, engine group, and month, respectively. Where λ_{ijk} is then the true engine in-flight shutdown rate for the air operator *i* and engine group *j* in month *k*. In this way, the problem of determining the engine in-flight shutdown rate of air operators boils down obtaining estimates of a set of Poisson parameters, which can be accomplished by using the maximum likelihood estimation method (MLE) or simultaneous estimate methods, i.e., Bayesian estimation methods.

The MLE estimates of the true engine in-flight shutdown rate is simply the number of shutdowns divided by the engine hours already divided by 1000. If all engines have nearly the same or identical true engine in-flight shutdown rate regardless of operating air operator and aircraft, one can then pool all engine shutdown data collected from air operators and obtain the MLE estimate of the true engine shutdown rate. Or, if there is enough evidence to accept that the engine in-flight shutdown rates are very different and totally unrelated, one can then obtain the MLE estimate using individual engine shutdown data. However, since the true engine in-flight shutdown rate is relatively a small number, the MLE estimates would be zero in most cases; a nonzero MLE estimate requires a large number of engine hours.

On the other hand, it is probably true that the engine in-flight shutdown data collected from air operators have some commonalties. This commonality is expected because air operators use the same or similar engines, and also, the fact that commercial engines share the same technology and, in most cases, are even manufactured by the same company. In literature, the Bayesian estimation methods as described, for example, in Maritz (1989) and Berger (1985), have been widely used to capture such commonality and provide simultaneous estimates of the parameters at the individual level.

Martz, Parker, and Rasmuson (1999) used a parametric empirical Bayes (PEB) approximation and a two-stage hierarchical Bayes (HB) formulation to estimate the scram rate for nuclear power plants. The annual number of unplanned scrams was assumed to follow a Poisson distribution. They assumed a conjugate gamma prior at the first stage and an improper distribution at the second stage. They developed the MLE estimates of scram rates as well as the HB and PEB estimates of posterior means and standard deviations for each plant and compared them. They then used an exponentially weighted moving average to smooth scram rate estimates over time in order to identify trends.

Schluter, Deely, and Nicholson (1997) developed a gamma-Poisson HB model to identify the most hazardous accident location. They considered informative and noninformative hyperpriors and discussed how to determine the type of prior densities and their parameters. The developed model is illustrated using fatality accidents data.

Martz, Kvam, and Abramson (1996) use a PEB approximation to estimate emergency diesel generator (EDG) reliability at the individual level using industrywide data. They assumed a binomial distribution model for the EDG reliability data and a beta prior distribution to develop the PEB estimates of the EDG's reliabilities. They proposed seven practical steps in applying the PEB methods.

Chen and Singpurwalla (1996) developed a two-stage HB model to obtain simultaneous estimates of binomial parameters and applied it to determine the EDG reliability using the Markov Chain Monte Carlo techniques under independence and exchangeability of the parameters, separately. They assumed an L-shaped prior distribution at the first stage and a beta distribution at the second. They then compared the results with those obtained by the maximum likelihood and empirical Bayes (EB).

Gaver and O'Muircheartaigh (1987) developed simultaneous estimate of Poisson parameters using the PEB method. They assumed two parametric prior distributions: log-student t and gamma distribution. They then analyzed three data sets and concluded that the individual estimates based on the student-t prior have a robust quality.

The first step in applying the Bayesian analysis to the engine in-flight shutdown data is to show that λ_{ijk}s are not the same for all *i*, *j*. The Chi-square test, as explained in Degroot (1986, p. 542), can be used to test the equality of λ_{ijk}s. However, since the number of engine in-flight shutdown in each month, in most cases, is zero or less than five, we let *k* represent a year and use annual data, as shown in Table 1, to apply the Chi-square test; each row in Table 1 corresponds to an air operator and engine group. The Chi-square statistic for the data in Table 1 is 1734.13 with 188 degrees of freedom and with a p-value of zero; however, we cannot reject the equality of λ_{ijk}s merely based on the result of this test since there are still plenty of engine in-flight shutdown data in Table 1 that is less than five. To provide more evidence against the equality of λ_{ijk} s, the Chi-square test is applied to subsets of the data in Table 1.

The Chi-square statistic for those that have at least five engine in-flight shutdowns is 1074.52 with 42 degrees of freedom and a p-value of zero. Moreover, the Chi-square statistic for major air operators (shown as in bold in Table 1) is 409.17 with 57 degrees of freedom and a p-value of zero. We notice that none of the above subsets are randomly chosen; however, zero p-values for all three tests strongly suggest that λ_{ijk} s are not identical for all *i*, *j*.

The *exchangeability* is another important issue that has to be investigated in a Bayesian analysis; a distribution is exchangeable if the probability distribution does not depend on the order of variables as described in Berger (1985). For a detail discussion on the exchangeability issue refer to the Draper, et al. (1993). In the case of engine shutdown events, we consider the distribution as exchangeable because one would attach the same probabilities to the different sequences of λ_{ijk} s.

Table 2 contains a turbojet's engine in-flight shutdown data for January 1999, and also contains the MLE, PEB, and HB Monte Carlo Markov Chain (MCMC) estimates that will be discussed in detail next.

2. Hierarchical Bayes Estimation

The HB or full Bayesian approach is based on the use of hierarchical priors, particularly two-stage priors. The engine in-flight shutdown problem is conceptually identical to the scram rate problem as described by Martz, et al. (1999). Thus, we similarly consider λ_{ijk} as a sample from the conjugate gamma distribution described as

$$\pi(\lambda_{ijk}|\alpha,\beta) = g(\lambda_{ijk}|\alpha,\beta)$$

where gamma distribution is defined as $g(x|\alpha,\beta) = \beta^{\alpha} x^{\alpha-1} \exp(-\beta x)/\Gamma(\alpha)$, $\alpha > 0, \beta > 0$. Therefore, the joint prior distribution for the data shown in Table 2 will then be as

$$\pi(\boldsymbol{\lambda}_k|\alpha,\beta) = \prod_{i=1}^{n}\prod_{j=1}^{n_i} g(\lambda_{ijk}|\alpha,\beta)$$

where $\boldsymbol{\lambda}_k$ is a vector including all λ_{ijk} s, k =1 (or January 1999), n and n_i are the number of air operators and the number of engine groups used by the air operator i, respectively. It is well known that the posterior distribution of $\boldsymbol{\lambda}_k$ will then be as

$$\pi(\boldsymbol{\lambda}_k|\alpha,\beta,\mathbf{x}_k) = \prod_{i=1}^{n}\prod_{j=1}^{n_i} g(\lambda_{ijk}|\alpha + x_{ijk}, \beta + T_{ijk})$$

where $\mathbf{x}_k$ is a vector including all x_{ijk} s. Note that the Bayes estimate of λ_{ijk} will be the mean of the posterior distribution, $\hat{\alpha}+x_{ijk} \big/ \hat{\beta}+T_{ijk}$, where $\hat{\alpha}$ and $\hat{\beta}$ are estimates of α and β, respectively.

At the second stage, the joint density function of prior parameters, $h(\alpha,\beta)$, can be written as

$$h(\alpha,\beta)=h_2(\beta|\alpha)h_1(\alpha)$$

We assume that $h_1(\alpha)$ follows an exponential distribution with a mean of 0.5 and $h_2(\beta|\alpha)$ follows a gamma distribution with a mean of 1 and standard deviation of 4, i.e., $h_2(\beta|\alpha)=g(\beta;0.0625,0.0625)$. We obtain HB estimates using Gibbs sampling as implemented in the Bayesian inference using Gibbs sampling software (BUGS), which is available at www.mrc-bsu.cam.ac.uk/bugs. Appendix A presents the BUGS language for the engine in-flight shutdown problem. The parameters of the posterior distribution for k=1 (or January 1999) are determined to be $\hat{\alpha}_1=0.3927$, $Var(\hat{\alpha}_1)=0.02468$, $\hat{\beta}_1=12.25$ and $Var(\hat{\beta}_1)=41.140$ after 10000 simulation iterations including 2000 burn-in iterations.

3. Parametric Empirical Bayes Estimate

The EB methods come to play when there is a known relationship among parameters, which allow determining the prior distribution using the data. The PEB models are those in which the known relationship is a parametric distribution. For the engine in-flight shutdown data, we have assumed that λ_{ijk}s follow a gamma distribution. Therefore, similar to Martz, et al. (1999), we use the marginal gamma-Poisson likelihood as

$$L(\alpha,\beta \mid \mathbf{x}_k) = \prod_{i=1}^{n}\prod_{j=1}^{n_i} \frac{\beta^{\alpha}\Gamma(\alpha + x_{ijk})T_{ijk}^{x_{ijk}}}{\Gamma(\alpha)\Gamma(1+x_{ijk})(\beta + T_{ijk})^{\alpha + x_{ijk}}}$$

to estimate α_k and β_k, which are the parameters of the prior distribution, i.e., gamma distribution. We use the Newton-Raphson and Fisher information matrix to obtain $\hat{\alpha}_k$, $\hat{\beta}_k$ and the variance-covariance matrix, respectively. The parameters of the prior are determined to be $\hat{\alpha}_1 = 0.3916$, $Var(\hat{\alpha}_1) = 0.0.02591$, $\hat{\beta}_1 = 12.22$, and $Var(\hat{\beta}_1) = 43.49$.

These estimates agree with those obtained using the HB approach. The MLE, PEB, and HB MCMC estimates of the engine in-flight shutdown rate obtained using data in Table 2 are plotted in Figure 1. The Bayes estimates, however, contain less variability compared with the MLE estimates. Furthermore, most of the MLE estimates are zero whereas the Bayes estimates are always nonzero. It is important to note that the true engine in-flight shutdown rate is greater than zero, and zero estimates, as obtained by the MLE method, are noninformative and useless.

The advantage of the PEB method over the HB is that the PEB method can be automated and used in developing a monitoring software for identifying air operators that are experiencing engine in-flight shutdown events at an extraordinary high rate. The exponentially smoothed PEB estimates of engine in-flight shutdown rate for air operator 1 (data item 1) and air operators as a group with a smoothing factor of 0.3 in 1999 are plotted in Figure 2. From this plot, air operator 1's performance regarding the engine in-flight shutdown events is quite well compared with the turbojet's group engine in-flight shutdown rate. Similar figures can be easily developed for the other air operators in Table 2.

4. Conclusion

We have obtained and compared the MLE, HB, and PEB estimates of the turbojet engine in-flight shutdown rate in January 1999. It is concluded that the Bayesian estimates are preferred over the MLE method in estimating the engine in-flight shutdown rate for monitoring purposes because they provide nonzero estimates and contain less variability. The exponentially smoothed Bayes estimates over time are also useful in identifying air operators with an extraordinary high engine in-flight shutdown rate.

Appendix A

```
model
	{
		for (i in 1 : N) {
			theta[i] ~ dgamma(alpha, beta)
			lambda[i] <- theta[i] * t[i]
			x[i] ~ dpois(lambda[i])
		}
		alpha ~ dexp(2)
		beta ~ dgamma(0.0625, 0.0625)
	}
list(t=c(95.226,187.827,57.350,1.232,14.164,11.554,16.722,13.586,3.360,1.392,22.740,2
6.550,6.862,26.378,9.324,9.512,14.488,8.950,0.620,3.795,0.090,1.545,32.838,89.808,57.
396,2.004,22.686,7.328,0,5.456,2.817,1.482,1.108,29.116,60.464,26.736,18.417,30.978,1
3.188,167.011,24.360,8.830,4.092,5.850,0.304,0.108,7.388,5.706,3.822,46.726,3.490,26.
169,2.530,1.064,0.448,1.440,4.408,5.691,9.752,7.010,8.022,3.560,5.308,4.772,3.594,0,8.
758,3.084,0.915,0.504,1.186,27.600,14.468,17.529,33.210,84.599,51.005,10.662,2.390,3.
644,0.492,6.548,0.012,2.794,4.176,0.394,0.786,1.362,1.638,0.714,3.082,11.226,1.964,1.6
53,1.401,2.346,10.110,6.094,0.348,0.528,139.432,19.520,4.382,1.440,0.153,5.094,10.002
,3.258,68.699,8.666,0.364,8.888,54.968,2.130,51.628,18.054,14.430,14.721,79.382,53.12
1,22.734,11.650,12.266,32.052,30.620,1.440,0.372,6.828,1.392,1.206,5.301,1.188,0.132,
1.254,0.998,0.308,19.890),
x=c(2,1,0,0,1,0,0,0,0,0,0,1,0,0,0,0,0,0,1,0,0,2,1,5,0,0,5,0,0,0,0,0,0,0,0,0,0,0,6,1,0,1,2,0,
0,0,0,1,0,0,1,0,0,0,0,3,1,1,0,0,0,0,1,0,0,1,1,0,0,0,0,1,0,0,2,2,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,2,0,0,4,0,0,0,0,0,0,0,1,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0),N=13
7)
list(alpha=0.1, beta=1)
```

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Figure 1- Estimates of the 01/1999 engine in-flight shutdown rate per 1000 engine hours

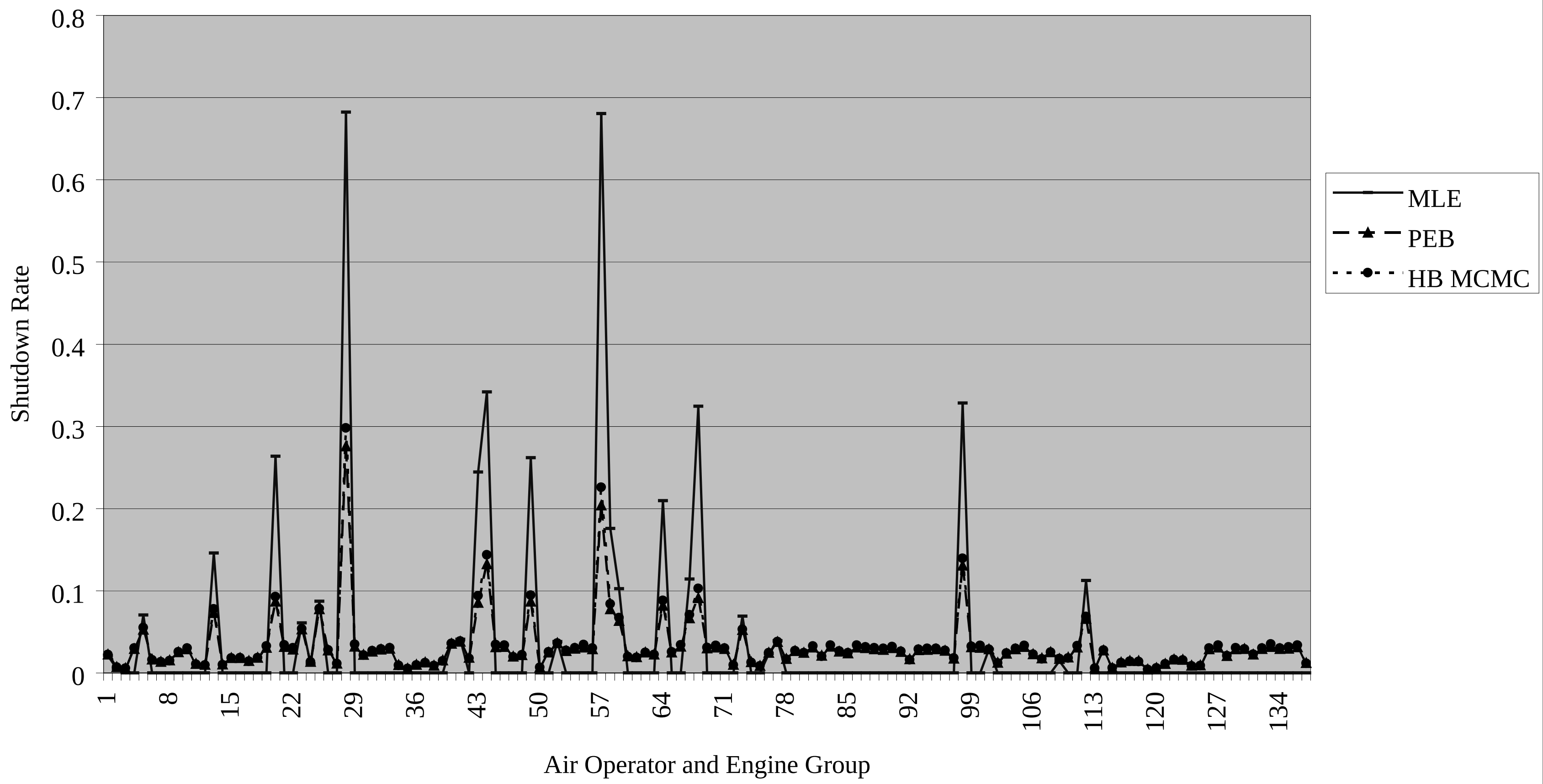

Figure 2- Exponentially Smoothed PEB estimates for Air Operator 1 (Data Item 1) in 1999

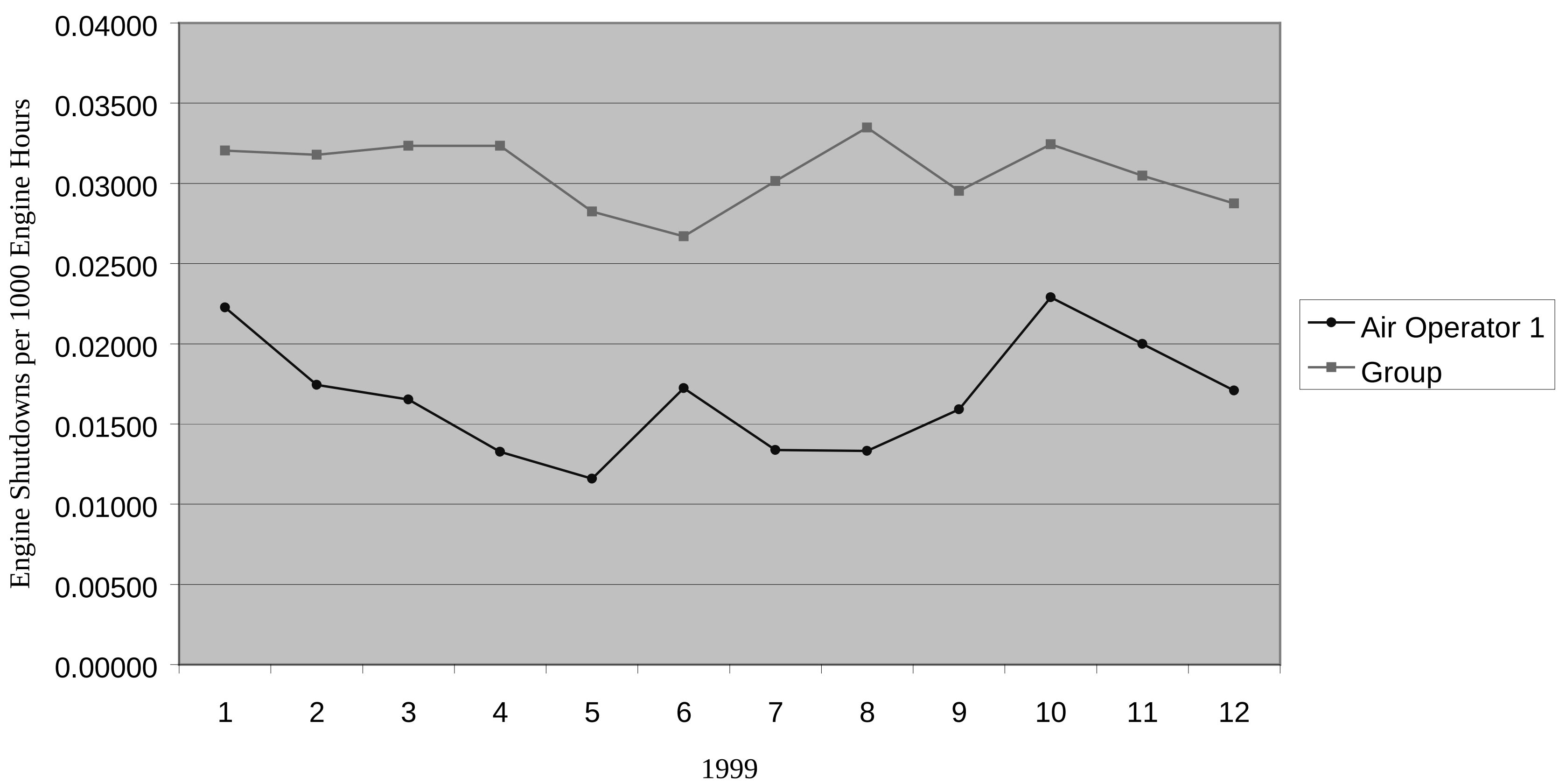

Table 1- The turbojet engine’s in-flight shutdown data for 1999

Hrs	SD	Hrs	SD	Hrs	SD	Hrs	SD
653945	**50**	124671	4	16548	1	14748	0
1909419	**49**	121572	4	13317	1	13236	0
1995791	**27**	12466	4	12747	1	13172	0
318078	25	**675280**	**3**	11922	1	12740	0
273616	**24**	**425200**	**3**	9036	1	12012	0
464514	**23**	**319374**	**3**	7995	1	11868	0
931480	**22**	214166	3	7798	1	11274	0
635980	**21**	208864	3	5073	1	9760	0
1056734	**19**	**207626**	**3**	4216	1	**9530**	**0**
53988	17	**197886**	**3**	2823	1	9069	0
153332	16	**183898**	**3**	2202	1	8739	0
120488	15	124884	3	**287216**	**0**	7581	0
140160	14	106140	3	**242370**	**0**	6927	0
1654956	**12**	100486	3	**232574**	**0**	6734	0
673620	12	75234	3	140304	0	6652	0
133820	12	62349	3	**137026**	**0**	6540	0
719934	**11**	43968	3	121582	0	6111	0
225674	**11**	37116	3	**98804**	**0**	5172	0
174964	11	**721280**	**2**	**97234**	**0**	4932	0
22456	11	**457764**	**2**	96750	0	4762	0
997566	**10**	**203452**	**2**	91714	0	**4664**	**0**
391384	10	**193280**	**2**	**88434**	**0**	**4590**	**0**
74472	10	**75378**	**2**	83324	0	4524	0
651447	**9**	61800	2	**77648**	**0**	4378	0
366310	9	56982	2	71662	0	4280	0
26766	9	53356	2	47344	0	3708	0
50516	8	51021	2	44604	0	3632	0
1055250	**7**	15784	2	41550	0	3446	0
657498	**7**	11724	2	**40272**	**0**	3304	0
209578	7	11616	2	**37884**	**0**	2922	0
179950	**7**	5004	2	**35716**	**0**	2832	0
52656	7	**395470**	**1**	**33178**	**0**	2788	0
52176	7	**341048**	**1**	32766	0	2152	0
201225	6	**226056**	**1**	29276	0	2064	0
141204	6	**218379**	**1**	27064	0	**2024**	**0**
99317	6	138792	1	26259	0	**1992**	**0**
958024	**5**	124058	1	25634	0	1326	0
697844	**5**	**113668**	**1**	24606	0	1202	0
76836	5	105639	1	22396	0	1034	0
70544	5	**100614**	**1**	21054	0	760	0
68886	5	77640	1	**20660**	**0**	736	0
49688	5	36840	1	20442	0	692	0
4632	5	31826	1	18681	0	348	0
345104	**4**	27444	1	18458	0	262	0
293871	**4**	20313	1	16965	0	34	0
245534	**4**	19566	1	16902	0		
178464	**4**	**17636**	**1**	16616	0		
135488	4	17248	1	14760	0		

Hrs: Hours, SD: Shutdown

Table 2- MLE and Bayes estimates of turbojet engine's in-flight shutdown rate for 01/1999

Data Item	Hours	Shutdowns	MLE	PEB		MCMC HB	
				Mean	Std Dev	Mean	Std Dev
1	95226	2	0.021003	0.02226	0.01439	0.02249	0.01440
2	187827	1	0.005324	0.00696	0.00590	0.00698	0.00594
3	57350	0	0	0.00563	0.00899	0.00563	0.00931
4	1232	0	0	0.02911	0.04653	0.03022	0.05381
5	14164	1	0.070602	0.05275	0.04472	0.05519	0.04831
6	11554	0	0	0.01647	0.02632	0.01587	0.02630
7	16722	0	0	0.01353	0.02162	0.01342	0.02229
8	13586	0	0	0.01517	0.02425	0.01509	0.02418
9	3360	0	0	0.02514	0.04017	0.02573	0.04532
10	1392	0	0	0.02877	0.04598	0.03020	0.05260
11	22740	0	0	0.01120	0.01790	0.01081	0.01784
12	26550	0	0	0.01010	0.01614	0.00957	0.01565
13	6862	1	0.14573	0.07294	0.06183	0.07797	0.07269
14	26378	0	0	0.01015	0.01621	0.00981	0.01630
15	9324	0	0	0.01818	0.02905	0.01817	0.03023
16	9512	0	0	0.01802	0.02880	0.01832	0.02993
17	14488	0	0	0.01466	0.02343	0.01417	0.02282
18	8950	0	0	0.01850	0.02956	0.01862	0.03148
19	620	0	0	0.03050	0.04874	0.03284	0.05740
20	3795	1	0.263505	0.08691	0.07367	0.09299	0.08392
21	90	0	0	0.03182	0.05084	0.03401	0.05971
22	1545	0	0	0.02845	0.04547	0.03042	0.05337
23	32838	2	0.060905	0.05308	0.03432	0.05355	0.03504
24	89808	1	0.011135	0.01364	0.01156	0.01335	0.01151
25	57396	5	0.087114	0.07745	0.03336	0.07823	0.03414
26	2004	0	0	0.02753	0.04400	0.02827	0.04996
27	22686	0	0	0.01122	0.01793	0.01147	0.01880
28	7328	5	0.682314	0.27585	0.11880	0.29810	0.15510
29	0	0	N/A	0.03205	0.05122	0.03489	0.06258
30	5456	0	0	0.02216	0.03541	0.02154	0.03554
31	2817	0	0	0.02604	0.04162	0.02687	0.04635
32	1482	0	0	0.02858	0.04568	0.02927	0.05089
33	1108	0	0	0.02938	0.04696	0.03046	0.05254
34	29116	0	0	0.00947	0.01514	0.00926	0.01490
35	60464	0	0	0.00539	0.00861	0.00548	0.00872
36	26736	0	0	0.01005	0.01606	0.00965	0.01603
37	18417	0	0	0.01278	0.02043	0.01247	0.02049
38	30978	0	0	0.00906	0.01449	0.00906	0.01530
39	13188	0	0	0.01541	0.02463	0.01461	0.02342
40	167011	6	0.035926	0.03566	0.01411	0.03587	0.01418
41	24360	1	0.041051	0.03804	0.03225	0.03887	0.03389
42	8830	0	0	0.01860	0.02973	0.01766	0.02898
43	4092	1	0.244379	0.08532	0.07233	0.09378	0.08661
44	5850	2	0.34188	0.13237	0.08559	0.14370	0.10150
45	304	0	0	0.03127	0.04997	0.03445	0.06185
46	108	0	0	0.03177	0.05077	0.03397	0.06224
47	7388	0	0	0.01997	0.03192	0.01974	0.03333
48	5706	0	0	0.02185	0.03491	0.02202	0.03624
49	3822	1	0.261643	0.08676	0.07355	0.09473	0.08905

Table 2 (Continue)- MLE and Bayes estimates of turbojet engine's in-flight shutdown rate for 01/1999

Data Item	Hours	Shutdowns	MLE	PEB		MCMC HB	
				Mean	Std Dev	Mean	Std Dev
50	46726	0	0	0.00664	0.01062	0.00666	0.01091
51	3490	0	0	0.02493	0.03984	0.02556	0.04388
52	26169	1	0.038213	0.03625	0.03073	0.03641	0.03148
53	2530	0	0	0.02655	0.04243	0.02749	0.04711
54	1064	0	0	0.02948	0.04712	0.03069	0.05365
55	448	0	0	0.03092	0.04941	0.03433	0.05970
56	1440	0	0	0.02867	0.04582	0.03009	0.05345
57	4408	3	0.680581	0.20400	0.11077	0.22580	0.14400
58	5691	1	0.175716	0.07770	0.06587	0.08393	0.07578
59	9752	1	0.102543	0.06334	0.05370	0.06735	0.06059
60	7010	0	0	0.02036	0.03254	0.01977	0.03307
61	8022	0	0	0.01935	0.03092	0.01897	0.03144
62	3560	0	0	0.02482	0.03966	0.02487	0.04313
63	5308	0	0	0.02234	0.03571	0.02255	0.03829
64	4772	1	0.209556	0.08191	0.06943	0.08807	0.08056
65	3594	0	0	0.02476	0.03958	0.02566	0.04325
66	0	0	N/A	0.03205	0.05122	0.03404	0.06255
67	8758	1	0.114181	0.06634	0.05624	0.07082	0.06275
68	3084	1	0.324254	0.09094	0.07709	0.10270	0.09650
69	915	0	0	0.02982	0.04765	0.03111	0.05707
70	504	0	0	0.03078	0.04919	0.03333	0.05866
71	1186	0	0	0.02921	0.04669	0.03033	0.05297
72	27600	0	0	0.00983	0.01572	0.00948	0.01547
73	14468	1	0.069118	0.05215	0.04421	0.05298	0.04501
74	17529	0	0	0.01316	0.02104	0.01283	0.02067
75	33210	0	0	0.00862	0.01377	0.00840	0.01352
76	84599	2	0.023641	0.02470	0.01597	0.02437	0.01591
77	51005	2	0.039212	0.03783	0.02446	0.03818	0.02506
78	10662	0	0	0.01711	0.02735	0.01671	0.02754
79	2390	0	0	0.02681	0.04284	0.02721	0.04672
80	3644	0	0	0.02469	0.03945	0.02447	0.04178
81	492	0	0	0.03081	0.04924	0.03281	0.05961
82	6548	0	0	0.02087	0.03335	0.02043	0.03309
83	12	0	0	0.03202	0.05117	0.03393	0.06105
84	2794	0	0	0.02608	0.04169	0.02679	0.04773
85	4176	0	0	0.02389	0.03817	0.02449	0.04047
86	394	0	0	0.03105	0.04962	0.03371	0.06219
87	786	0	0	0.03011	0.04812	0.03167	0.05650
88	1362	0	0	0.02884	0.04608	0.03062	0.05298
89	1638	0	0	0.02826	0.04516	0.02988	0.05218
90	714	0	0	0.03028	0.04839	0.03196	0.05613
91	3082	0	0	0.02559	0.04090	0.02638	0.04408
92	11226	0	0	0.01670	0.02669	0.01606	0.02660
93	1964	0	0	0.02761	0.04412	0.02884	0.05123
94	1653	0	0	0.02823	0.04511	0.02977	0.05313
95	1401	0	0	0.02875	0.04595	0.02980	0.05151
96	2346	0	0	0.02689	0.04297	0.02755	0.04623
97	10110	0	0	0.01754	0.02803	0.01813	0.03070
98	6094	2	0.328192	0.13061	0.08445	0.13960	0.09889

Table 2 (Continue)- MLE and Bayes estimates of turbojet engine's in-flight shutdown rate for 01/1999

Data Item	Hours	Shutdowns	MLE	PEB		MCMC HB	
				Mean	Std Dev	Mean	Std Dev
99	348	0	0	0.03116	0.04980	0.03238	0.05902
100	528	0	0	0.03072	0.04910	0.03337	0.06035
101	139432	4	0.028688	0.02896	0.01382	0.02888	0.01376
102	19520	0	0	0.01234	0.01972	0.01203	0.02009
103	4382	0	0	0.02359	0.03770	0.02409	0.04183
104	1440	0	0	0.02867	0.04582	0.03004	0.05410
105	153	0	0	0.03165	0.05058	0.03356	0.06054
106	5094	0	0	0.02262	0.03615	0.02269	0.03854
107	10002	0	0	0.01762	0.02816	0.01701	0.02862
108	3258	0	0	0.02530	0.04044	0.02541	0.04411
109	68699	1	0.014556	0.01720	0.01458	0.01731	0.01471
110	8666	0	0	0.01875	0.02996	0.01823	0.03045
111	364	0	0	0.03112	0.04974	0.03293	0.06172
112	8888	1	0.112511	0.06593	0.05589	0.06850	0.06178
113	54968	0	0	0.00583	0.00931	0.00586	0.00946
114	2130	0	0	0.02729	0.04361	0.02763	0.04734
115	51628	0	0	0.00613	0.00980	0.00607	0.00984
116	18054	0	0	0.01294	0.02067	0.01248	0.02065
117	14430	0	0	0.01469	0.02348	0.01417	0.02342
118	14721	0	0	0.01454	0.02323	0.01392	0.02353
119	79382	0	0	0.00427	0.00683	0.00427	0.00704
120	53121	0	0	0.00599	0.00958	0.00595	0.00988
121	22734	0	0	0.01120	0.01790	0.01112	0.01813
122	11650	0	0	0.01641	0.02622	0.01639	0.02676
123	12266	0	0	0.01599	0.02556	0.01572	0.02591
124	32052	0	0	0.00884	0.01414	0.00864	0.01417
125	30620	0	0	0.00914	0.01461	0.00887	0.01454
126	1440	0	0	0.02867	0.04582	0.03025	0.05388
127	372	0	0	0.03110	0.04970	0.03391	0.06105
128	6828	0	0	0.02056	0.03286	0.02087	0.03516
129	1392	0	0	0.02877	0.04598	0.03042	0.05470
130	1206	0	0	0.02917	0.04662	0.02910	0.05070
131	5301	0	0	0.02235	0.03572	0.02289	0.03916
132	1188	0	0	0.02921	0.04668	0.03036	0.05095
133	132	0	0	0.03171	0.05067	0.03526	0.06296
134	1254	0	0	0.02907	0.04645	0.03023	0.05345
135	998	0	0	0.02963	0.04735	0.03169	0.05766
136	308	0	0	0.03126	0.04996	0.03392	0.06469
137	19890	0	0	0.01220	0.01949	0.01176	0.01889